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Interaction of Two Pulses in Defocusing Nonlinear Schrodinger Equation

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Abstract. In this work we are using initial conditions in the form of two separated bright pulses with the rectangular shape to analyse the interaction of pulses in a defocusing Nonlinear Scrödinger Equation (NLSE). By exact solution of the direct scattering problem associated with the defocusing NLSE, we have obtained the expressions for long time behavior of the solution. Obtained results interpreted as a nonlinear interference of two pulses.

1. Introduction

Interaction of a beams and pulses is considered as one of the most important problems of the wave theory and has many applications in physics and engineering. This problem is well investigated in a small perturbation approximation, when an equations describing process of interaction are linear, and the developed theory explains the interference phenomena [1]. Recently, the study of an interaction in nonlinear media attracted much attention of researches [2], especially in optics, physics of Bose Einstein condensates, plasma physics, hydrodynamics etc. One of the generic equations of nonlinear wave theory is the Nonlinear Schrodinger equation (NLSE), which describes the propagation of an envelope of modulated wave in weekly nonlinear dispersive media [2]. This equation in one dimensional case is solvable by inverse scattering transform (IST) method, which allows to find different solutions of NLSE. By this method the bright soliton solutions and interaction of pulses were investigated in details for focusing NLSE [3]. The interaction of two pulses of the defocusing NLSE was considered in [4] with the help of Inverse Scattering Transform method. Obtained results were applied to explain some features of the nonlinear interference in Bose-Einstein condensates observed in experiments [5]. Recent development in this area presented in the review paper [6]. In the paper [4] the initial condition is chosen in the Gaussian form. This choice is not allowed to obtain explicit expression for the description of the interference patterns. This circumstance has compelled authors to use the approximate methods in deriving of final results. In this work we are using initial conditions in the form of two separated bright pulses with the rectangular shape. This choice allows us to study the important features of a nonlinear interference. By exact solution of the direct scattering problem associated with the defocusing NLSE, we have obtained the expressions for long time behavior of the solution.

2. Formulation of the problem

Following [4] we will write the defocusing NLSE in scaled form

$$iq_\tau + \frac{1}{2}q_{xx} - g|q|^2q = 0. \tag{1}$$

Let us first rewrite the Eq.(1) in the more convenient for calculations form

$$iq_\tau + \frac{1}{2}q_{xx} - 2|q|^2q = 0. \tag{2}$$

where we have used the following change of variables $t \to t/2$, $x \to x$, $q \to \sqrt{g}q$.

The equation (2) is completely integrable [7], that is it can be presented as a compatibility condition of two linear systems $\psi_x = U\psi$, $\psi_t = V\psi$, where U and V are 2×2 matrices.

$$U = i\lambda \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$$

$$V = -2i\lambda^2 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - 2\lambda \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix} - i \begin{pmatrix} 0 & q_x \\ -q_x^* & 0 \end{pmatrix} + i \begin{pmatrix} |q|^2 & 0 \\ 0 & -|q|^2 \end{pmatrix}.$$

The eigenvalue Zakharov-Shabat problem for the auxiliary two-component eigenfunction $(\psi_1, \psi_2)^T$ is written as the following system

$$\frac{\partial \psi_1}{\partial x} = -i\lambda\psi_1 + iq(0,x)\psi_2, \tag{3}$$

$$\frac{\partial \psi_2}{\partial x} = i\lambda\Psi_1 + iq^*(0,x)\psi_1. \tag{4}$$

It is well known that the defocusing NLSE (2), does not have the bright soliton solutions, it follows from the fact that the system (3-4) does not have discrete eigenvalues. Any initial localized pulse on zero background will disperse and eventually disappear. Our aim is to consider the interaction of two localized pulses with the objective to discuss the influence of nonlinearity to interference process. The long time asymptotic solution of Eq(2) can be found by calculation of the transmission coefficient $a(\lambda)$ for system (3-4), considering $q(x,0)$ as the potential. The expression for intensity $|q|^2$ in this case has the following form [8]

$$|q|^2 = \frac{}{1/4\pi t} \ln |a(\lambda = -x/4t)|^2. \tag{5}$$

3. Solution for two-box initial conditions

Let us now to concretize the initial conditions which we are going to consider for Eq(2). Initial conditions are two rectangular pulses:

$$q(x,0) = q_{10}B(x,x_1,x_2)\exp(2i\nu_1 x) + q_{20}B(x,x_3,x_4)\exp(2i\nu_2 x), \tag{6}$$

where q_{10} and q_{20} are constant amplitudes, $x_1 < x_2 < x_3 < x_4$, $x_3 - x_2$ is the separation distance, $d_1 = x_2 - x_1$ and $d_2 = x_4 - x_3$ are the widths of each of the pulses , $B(y, y_1, y_2) = [\theta(y - y_1) - \theta(y - y_2)]$, and θ is the Heavyside step function.

We will try now to solve the system (3-4), considering $q(x, 0)$ formally as the external potential. Our approach is to find solutions in different regions and then use the continuity conditions. In the domains where the potential equals to zero the solution is

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} A\exp(-i\lambda x) \\ B\exp(i\lambda x) \end{pmatrix}, \tag{7}$$

where A and B are some unknown coefficients.

Let us represent the potential in the nonzero regions in the following general form $q = q_0 \exp(i\nu x)$. To find the solutions in the regions with nonzero potential, we will transform the system (3-4) to the following scalar second order ordinary differential equation for v

$$v_{xx} + \xi^2 v = 0 \tag{8}$$

where $\lambda + \nu = k$, $(\lambda + \nu)^2 - |q|^2 = \xi^2$ and the new variable v defined by the equation $\psi_1 = v\exp(i\nu x)$.

Solution of the Eq.(8) is

$$v = \alpha_1 e^{-i\xi x} + \alpha_2 e^{i\xi x},$$

so the solution for ψ_1 will be

$$\psi_1 = \alpha e^{-i(\xi-\nu)x} + \delta e^{i(\xi+\nu)x}.$$

By the same way we can find solution for ψ_2

$$\psi_2 = \sigma e^{-i(\xi-\nu)x} + \beta e^{i(\xi+\nu)x}.$$

Solutions for ψ_1, ψ_2 are overdetermined there are four constants, instead two. One can determine the first pair through second as follows

$$\sigma = -\frac{k-\xi}{iq_0}\alpha,$$
$$\delta = \frac{k-\xi}{iq_0^*}\beta.$$

So we have

$$\psi_1 = \alpha e^{-i(\xi-\nu)x} + \frac{(k-\xi)\beta}{iq_0^*} e^{i(\xi+\nu)x}, \tag{9}$$

$$\psi_2 = -\frac{(k-\xi)\alpha}{iq_0} e^{-i(\xi-\nu)x} + \beta e^{i(\xi+\nu)x}. \tag{10}$$

Now using formulas (7) and(9-10) one is able to write the solutions in the different domains.

I. We are considering the scattering problem for system (3-4), so in the domain $x < x_1$ we have only the initial plane wave with unit amplitude

$$\Psi = \begin{pmatrix} \exp(-i\lambda x) \\ 0 \end{pmatrix}. \tag{11}$$

II. $x_1 < x < x_2$

$$\psi_1 = \alpha_1 e^{-i(\xi_1-\nu_1)x} + \frac{(k_1-\xi_1)\beta_1}{iq_{10}^*} e^{i(\xi_1+\nu_1)x}, \tag{12}$$

$$\psi_2 = -\frac{(k_1-\xi_1)\alpha_1}{iq_{01}} e^{-i(\xi_1-\nu_1)x} + \beta_1 e^{i(\xi_1+\nu_1)x}. \tag{13}$$

III. $x_2 < x < x_3$

$$\Psi = \begin{pmatrix} C\exp(-i\lambda x) \\ D\exp(i\lambda x) \end{pmatrix}. \tag{14}$$

IV. $x_3 < x < x_4$

$$\psi_1 = \alpha_2 e^{-i(\xi_2-\nu_2)x} + \frac{(k_2-\xi_2)\beta_2}{iq_{20}^*} e^{i(\xi_2+\nu_2)x}, \tag{15}$$

$$\psi_2 = -\frac{(k_2-\xi_2)\alpha_2}{iq_{02}} e^{-i(\xi_2-\nu_2)x} + \beta_2 e^{i(\xi_2+\nu_2)x}. \tag{16}$$

V. $x_4 < x$

$$\Psi = \begin{pmatrix} a\exp(-i\lambda x) \\ b\exp(i\lambda x) \end{pmatrix}. \tag{17}$$

The next step is to use continuity conditions and to fit the solution at the boundary points between different domains. After some simple but lengthly calculations one is able to obtain explicit expressions for the coefficients C, D, α_1, β_1, α_2, β_2, a and b. We will right below only the formula for a

$$a(\lambda) = \exp[i(k_1d_1+k_2d_2)]\{[\cos(\xi_1d_1) - i\frac{k_1}{\xi_1}\sin(\xi_1d_1)][\cos(\xi_2d_2) - i\frac{k_2}{\xi_2}\sin(\xi_2d_2)] + \frac{q_{10}q_{20}}{\xi_1\xi_2}\exp[2i(-k_1x_2+k_2x_3)]\sin(\xi_1d_1)\sin(\xi_2d_2)\} \tag{18}$$

4. Discussion and conclusions

The expression for long time asymptotic solutions for intensity of Eq.(1) can be now written by returning to the original variables and substituting expression for $a(\lambda)$ to Eq.(5). Analysis of this expression shows that the asymptotic behavior is as expected dispersing with the clear interference fringes. The account of nonlinearity changes the distribution of the fringes and amplitude of the field, and should be taken into account. Obtained formulas may help in explaining results of interference experiments in one-dimensional nonlinear systems and also can find applications in applied interferometry for determining a materials properties from an interference experiments.

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